456/1
MATHEMATICS
Paper 1
July/Aug. 2018
$2^1/_2$ hours

''Together for Mathematics''

SECONDARY MATHEMATICS TEACHERS' ASSOCIATION

SMATA JOINT MOCK EXAMINATIONS 2018
Uganda Certificate of Education

MATHEMATICS

Paper 1

2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES:

Answer **all** questions in Section **A** and any **five** questions from Section **B**.

Any additional question(s) answered will **not** be marked.

3

All necessary calculations **must** be done on the same answer booklet provided. Therefore, no paper should be given for rough work.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

State the **degree of accuracy** at the end of each question attempted using calculator or mathematical table and indicate **Cal** for calculator or **Tab** for mathematical tables.

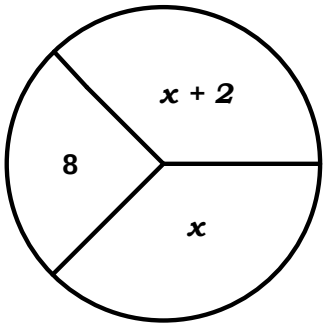
SECTION A : (40 MARKS)

Answer **all** the questions in this section.

1. Make M the subject of the formula, $Q = \sqrt{\frac{3B}{N-M}}$. ***(04 marks)***

2. Matrix $P = \begin{pmatrix} 3 & -1 \\ 2 & 1 \end{pmatrix}$ and $Q = \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}$. Find the matrix R such that $P^2 = PQ - R$. ***(04 marks)***

3. In the diagram below, the angle representing $x + 2$ is 120^o. Find the value of x. ***(04 marks)***



4. The length of a rectangular carpet is 4 metres more than its width. If its area is $12m^2$, find the width of the carpet. ***(04 marks)***

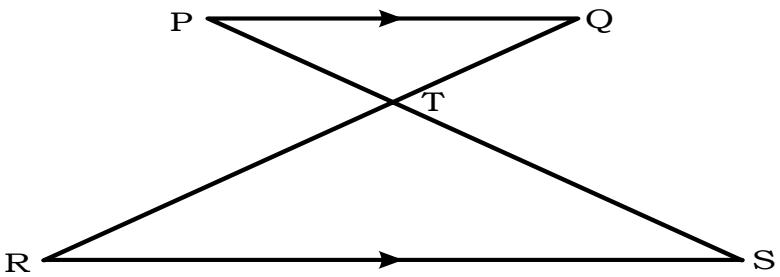
5. Find the equation of a straight line which passes through the point A (-1, 2) and it is parallel to the line $x - 2y - 6 = 0$. ***(04 marks)***

6. Given that $13\sin\theta + 5 = 0$ for $100^o \leq \theta \leq 270^o$, find the value of $\cos\theta + \tan\theta$. ***(04 marks)***

7. Use matrix method to solve the equations;
   $\frac{1}{2}x + y = \frac{5}{2}$ and $x + \frac{2}{3}y = \frac{7}{3}$ ***(04 marks)***

8. The mean of n, 8n+1, 17 and 20 is 14.
   Find the;
   (i) Value of n
   (ii) Mode of the data. ***(04 marks)***

9. In the figure below, PQ is parallel to RS and the lines PS and RQ meet at T
   Given that PT: TS = 2:3 and that RQ =10cm, find the length of RT. ***(04 marks)***



10. Given that $a * b = ab - b^2$. Find $(4 * -1) * (2 * 1)$. ***(04 marks)***

SECTION B: (60 MARKS)

Attempt any **five** questions from this section

11. The data below shows the weights in kg of fish trapped in River Kagera.

Weights (kg)	4.8 – 5.2	5.3 – 5.7	5.8 – 6.2	6.3 – 6.7	6.8 – 7.2
Number of fish	3	8	12	10	7

(a) State the;
  (i) Class width
  (ii) Median class

(b) Calculate the mean weight.

(c) Draw a histogram and hence use it to estimate the modal weight.

(12 marks)

12. Given the transformations represented by matrices:

$T_1 = \begin{pmatrix} 2 & 1 \\ -1 & -2 \end{pmatrix}$ and $T_2 = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$

T_1 followed by T_2 can be represented by a single matrix T.

(a) Find the matrix T and its inverse.

(b) The points $A''(7,-11)$, $B''(-7,-13)$and $C^{11}(-8, 16)$ are images of A, B and C under T. Find the coordinates of A, B and C.

(c) Find the coordinates of $A'B'$ and C' the image of A, B and C under the transformation T_1. ***(12 marks)***

13. Using a ruler and a pair of compasses only,

(a) Construct a parallelogram ABCD such that AB = 6cm, BC = 4.8cm and angle ABC = 150^o.

(b) Construct a perpendicular from D to meet AB at M. measure the length DM.

(c) Draw a circle through the points M, A and D. Measure the radius of the circle. ***(12 marks)***

14. (a) Draw a graph of $y = 2x^2 - x - 3$ for $-3 \leq x \leq 3$.
Use scales of 2cm to represent 1 unit on the x-axis and 1cm to represent 1 unit on the y-axis.

(b) On the same axes, draw the line $y = x + 1$.

(c) (i) Use the two graphs to solve the equation $x^2 - x - 2 = 0$

(ii) Find the minimum value of the function $y = 2x^2 - x - 3$

(12 marks)

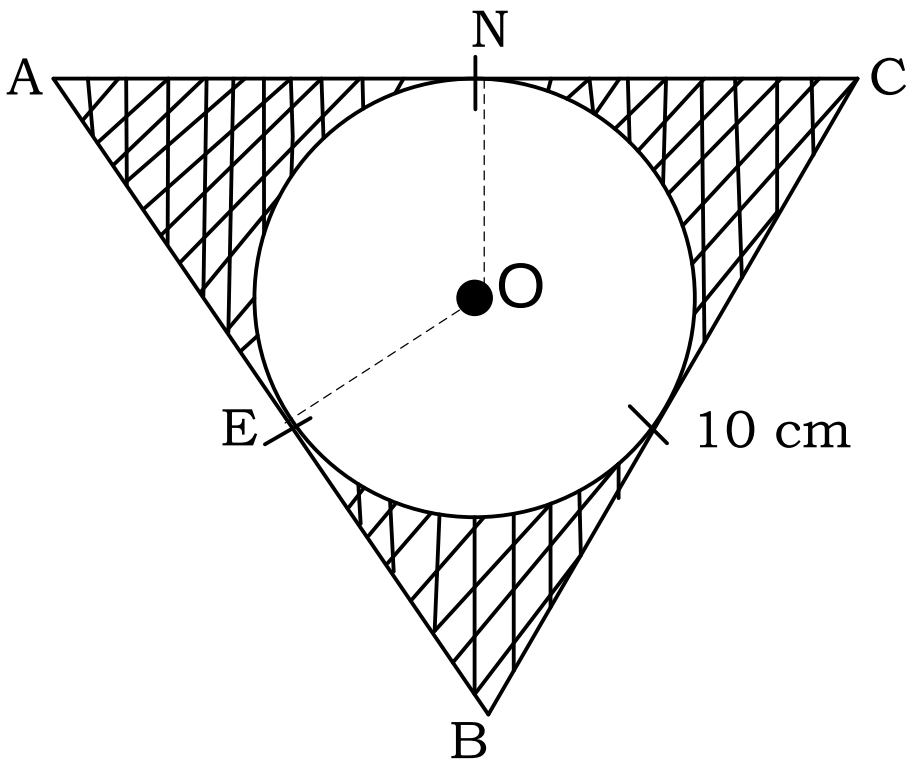
15. A garden measures 10m by 8m. A uniform path is made all round the garden. The total area of the path is $88m^2$.

(a) Find the width of the path.

(b) The path in (a) above is to be covered by a square concrete slab. Each corner of the path is covered by small slabs whose side is equal to the width of the path and rest of the path is covered by small slab of side 50cm each. Find the number of slabs required.

(c) The cost of making each corner slab is shs. 6,000 and each smaller slab is shs. 5,000. Calculate the cost of covering the path. ***(12 marks)***

16. The diagram below shows an inscribe circle within a triangle ABC.



Given that triangle ABC is equilateral measuring 10cm and O is the centre of the circle, find:

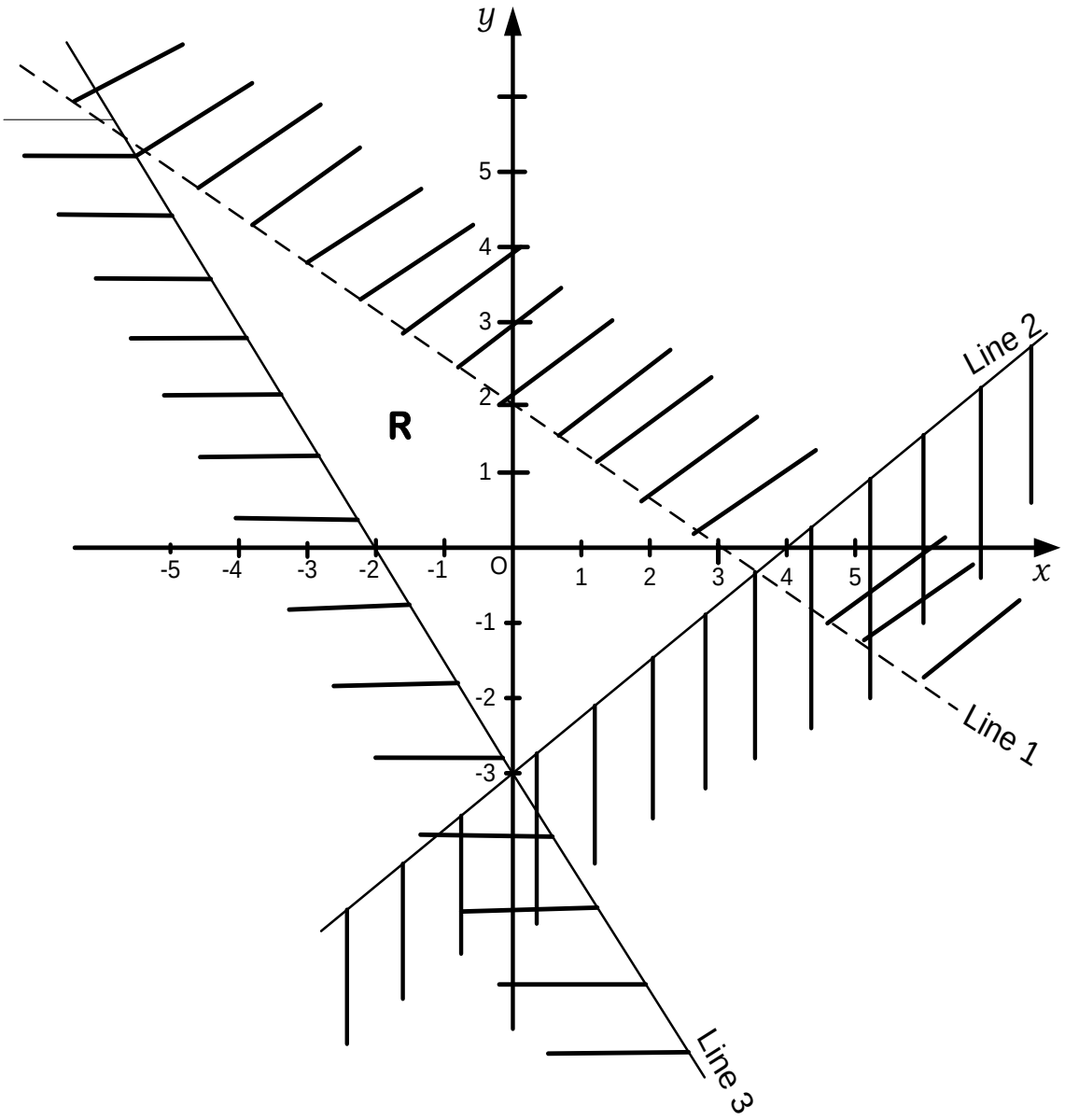
(i) the radius of the circle.

(ii) the area of the sector EON

(iii) Area of the shaded region. ***(12 marks)***

17. (a) Solve and list all the integral values of x which satisfy the inequalities $x + 6 \leq 15 - 2x < 2x + 21$ ***(04 marks)***

(b) The figure shows a graph of inequalities with shaded unwanted area.

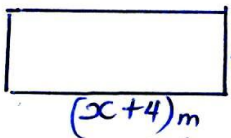


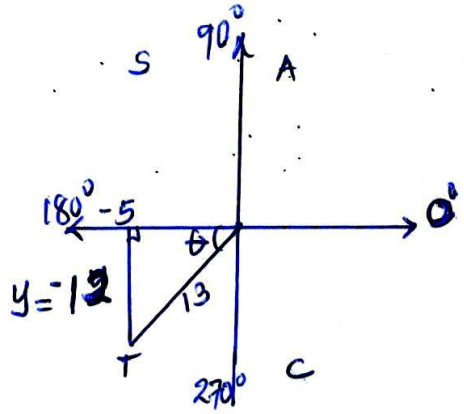
Write down all the inequalities represented by the region R. ***(08 marks)***

END

SMATA JOINT MOCK MATHEMATICS EXAMS 2018
CLASS: S.4 **PAPER:** I **PAPER CODE:** 456/1

No.	SOLUTION	MARKS	COMMENTS
1.	$Q^2 = \left(\sqrt{\frac{3B}{N-M}}\right)^2$	M_1	Squaring both sides
	$Q^2 = \frac{3B}{N-M}$		
	$Q^2(N-M) = 3B$	M_1	Cross multiplying
	$N-M = \frac{3B}{Q^2}$	M_1	Dividing through by Q^2.
	$M = N - \frac{3B}{Q^2}$	A_1	
	$\therefore M = \frac{NQ^2 - 3B}{Q^2}$		Any of the two forms
		04	
2.	$\begin{pmatrix} 3 & -1 \\ 2 & 1 \end{pmatrix}^2 = \begin{pmatrix} 3 & -1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix} - R$	M_1	
	$\begin{pmatrix} 3 & -1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 3 & -1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 6+1 & 9-1 \\ 4-1 & 6+1 \end{pmatrix} - R$		
	$\begin{pmatrix} 7 & -4 \\ 8 & -1 \end{pmatrix} = \begin{pmatrix} 7 & 8 \\ 3 & 7 \end{pmatrix} - R$	B_1	
	$R = \begin{pmatrix} 7 & 8 \\ 3 & 7 \end{pmatrix} - \begin{pmatrix} 7 & -4 \\ 8 & -1 \end{pmatrix}$	M_1	
	$\therefore R = \begin{pmatrix} 0 & 12 \\ 11 & 6 \end{pmatrix}$	A_1	
		04	

No.	SOLUTION	MARKS	COMMENTS
3-	Total $= x+2+x+8 = 2x+10$	M_1	
	$\frac{x+2}{2x+10} \times 360^\circ = 120^\circ$	M_1	
	$\frac{x+2}{2x+10} = \frac{1}{3}$		
	$3x+6 = 2x+10$	M_1	
	$\therefore x = 4$	A_1	
		04	
4-			
	Let Width of rectangle $= x$		
	Area $= (x+4)x$		
	$\Leftrightarrow x^2+4x = 12$	M_1	
	$x^2+4x-12 = 0$		
	$(x-2)(x+6) = 0$	M_1	
	Either $x-2 = 0$		
	$x = 2$		
	or $x+6 = 0$		
	$x = -6$	B_1	for both values of x
	Discarding $x = -6$		
	$\Leftrightarrow x = 2m$		
	$\therefore$ The width is 2m	A_1	For accurate value of x
		04	

No.	SOLUTION	MARKS	COMMENTS
5.	(i) Vowels = {I, E, U, A, I, I, E}	B1	
	No. of letters = 12		
	$P(\text{a vowel}) = \frac{n(E)}{n(S)} = \frac{7}{12}$	A1	
	(ii) $P(\text{a T or E}) = P(T) + P(E)$		
	$= \frac{1}{12} + \frac{2}{12}$	M1	
	$= \frac{3}{12}$		
	$= \frac{1}{4}$	A1	
		04	
6.	$\sin\theta = \frac{-5}{13}$		
			
	$y = \sqrt{13^2 - (-5)^2}$	B1M1	B1 - for the sketch
	$y = \sqrt{169 - 25}$		M1 - for applying the theorem
	$y = \sqrt{144}$		
	$\therefore y = 12$		
	$\cos\theta = \frac{-5}{13}$, $\tan\theta = \frac{-12}{-5}$		
	$= \frac{12}{5}$	M1	
	$\cos\theta + \tan\theta = \frac{-5}{13} + \frac{12}{5}$		
	$= \frac{131}{65}$	A1	
		04	

No.	SOLUTION	MARKS	COMMENTS
7.	$x + 2y = 5$ ---(i) $3x + 2y = 7$ ---(ii) $\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$ $\begin{pmatrix} 2 & -2 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & -2 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 5 \\ 7 \end{pmatrix}$ $\begin{pmatrix} -4 & 0 \\ 0 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 + -14 \\ -15 + 7 \end{pmatrix}$ $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \frac{-1}{4}\begin{pmatrix} -4 \\ -8 \end{pmatrix}$ $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $\therefore x = 1$ and $y = 2$	M_1 M_1 A1 A1	for x and y
		04	
8.	(i) $\frac{n + 8n + 1 + 17 + 20}{4} = 14$ $9n + 38 = 56$ $\frac{9n}{9} = \frac{18}{9}$ $\therefore n = 2$	M_1 A1	
	(ii) 2, 17, 17, 20	B1	
	$\therefore$ The mode is 17	B1	
		04	

No.	SOLUTION	MARKS	COMMENTS
9.	$a * b = ab - b^2$ $(4 * -1) = 4(-1) - (-1)^2 = -4 - 1$ $= -5$ $(2 * 1) = 2 \times 1 - (1)^2$ $= 2 - 1$ $= 1$ $(4 * -1) * (2 * 1) = -5 * 1$ $-5 * 1 = -5 \times 1 - (1)^2$ $= -5 - 1$ $\therefore (4 * -1) * (2 * 1) = -6$	B1 B1 M1 A1	
		04	
10.	An enlargement with centre O and scale factor K is $\begin{pmatrix} K & 0 \\ 0 & K \end{pmatrix}$ Transformation x object = Image $\begin{pmatrix} K & 0 \\ 0 & K \end{pmatrix}\begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} -6 \\ -15 \end{pmatrix}$ $2K + 0 = -6$ $2K = -6$ $K = -3$ or $0 + 5K = -15$ $\therefore K = -3$ $\therefore$ Matrix is $\begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}$	M1 M1 A1 B1	
		04	

No.	SOLUTION	MARKS	COMMENTS

11.

Weight (Kg)	f	x	fx	Boundaries
4·8 – 5·2	3	5	15	4·75 – 5·25
5·3 – 5·7	8	5·5	44	5·25 – 5·75
5·8 – 6·2	12	6	72	5·75 – 6·25
6·3 – 6·7	10	6·5	65	6·25 – 6·75
6·8 – 7·2	7	7	49	6·75 – 7·25
TOTAL	40	~~30~~	245	

M1 Column of x

M1 Column of fx

A1 for Σfx

B1 for labelling axes

M2 for all correct bars

M1 for 3 to 4 bars correct

M1 For Crossings on tallest bar

B1

(a) (i) Class width = 0·5

B1

(ii) Median class is 5·8 – 6·2

B1

(b) Mean weight $= \dfrac{\Sigma fx}{\Sigma f}$

$= \dfrac{245}{40}$

M1 √ for his Σfx

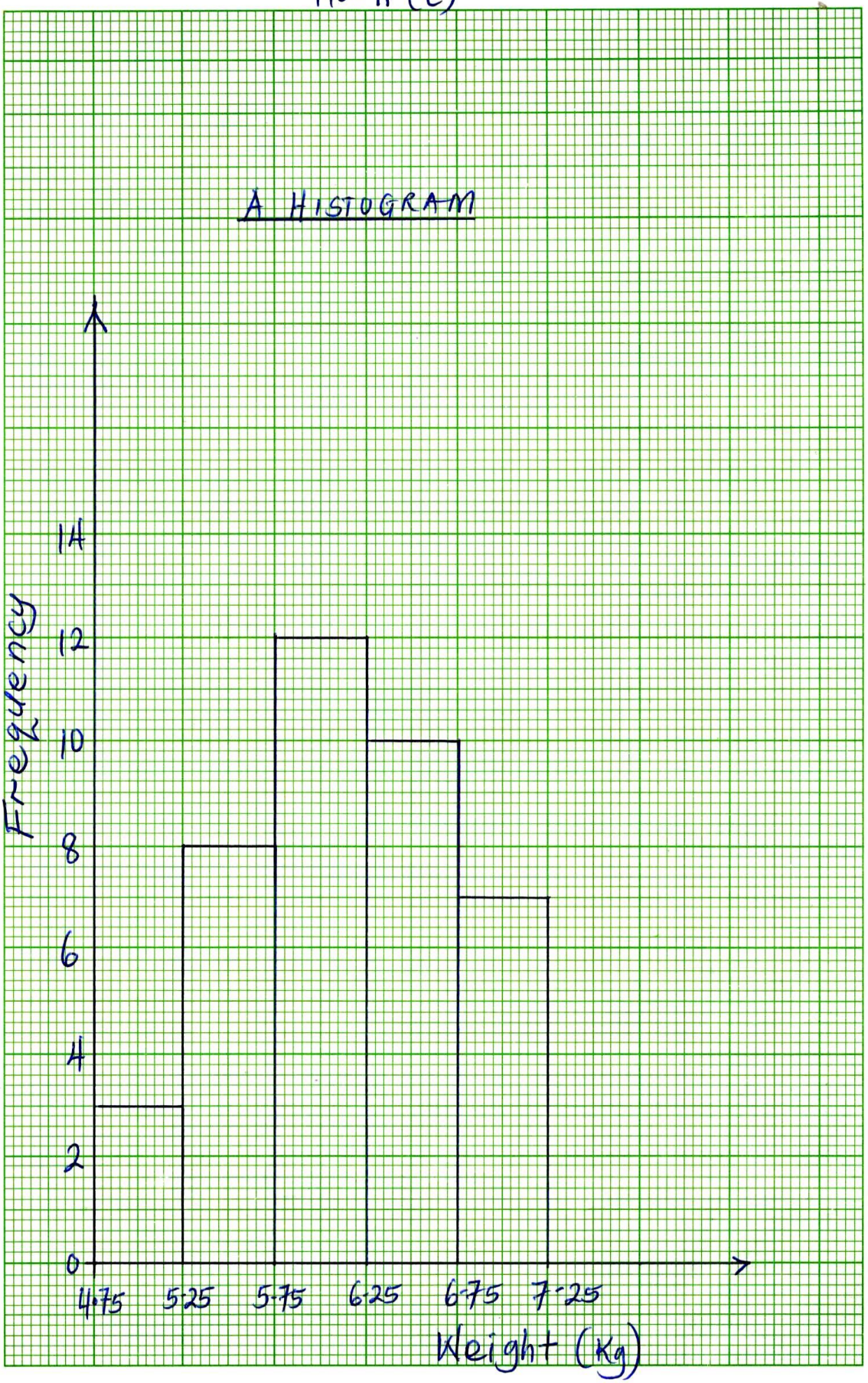
$= 6.125$ Kg

A1

(c) Modal weight = 6·05 – 6·15 Kg

A1 For correct modal weight

No. 11 (c)

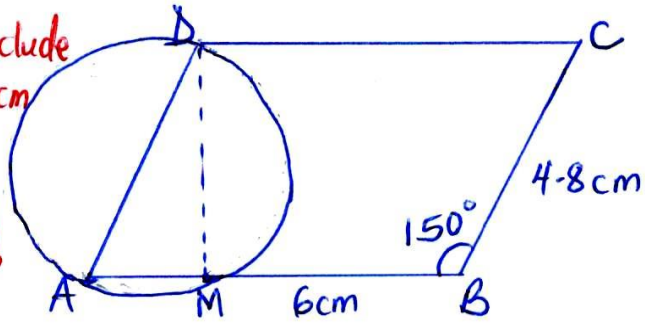


No.	SOLUTION	MARKS	COMMENTS
12.	$T_1 = \begin{pmatrix} 2 & 1 \\ -1 & -2 \end{pmatrix}$, $T_2 = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$		
	(a) $T = T_2 T_1$		
	$= \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ -1 & -2 \end{pmatrix}$	M_1	
	$= \begin{pmatrix} 6+-1 & 3+-2 \\ 2+-3 & 1+-6 \end{pmatrix}$		
	$T = \begin{pmatrix} 5 & 1 \\ -1 & -5 \end{pmatrix}$	B_1	
	$\det(T) = -25 - -1$ $= -24$		
	$T^{-1} = \frac{1}{-24}\begin{pmatrix} -5 & -1 \\ 1 & 5 \end{pmatrix}$	M_1	Adjunct & Det
	$= \begin{pmatrix} 5/24 & 1/24 \\ -1/24 & -5/24 \end{pmatrix}$	A_1	
	(b) $T^{-1} \times I = \begin{pmatrix} 5/24 & 1/24 \\ -1/24 & -5/24 \end{pmatrix}\begin{pmatrix} 7 & -7 & -8 \\ -1 & -13 & 16 \end{pmatrix}$	M_1	
	$= \begin{pmatrix} 17/12 & -2 & -1 \\ -1/12 & 3 & -3 \end{pmatrix}$		
	$\therefore A\left(\frac{17}{12}, -\frac{1}{12}\right), B(-2, 3), C(-1, -3)$	$A_1 A_1 A_1$	

No.	SOLUTION	MARKS	COMMENTS
12. (c)	Transformation × obj = Image $= \begin{pmatrix} 2 & 1 \\ -1 & -2 \end{pmatrix} \begin{pmatrix} 17/12 & -2 & -1 \\ -1/2 & 3 & -3 \end{pmatrix}$ $= \begin{pmatrix} 11/4 & -1 & -5 \\ -5/4 & -4 & 7 \end{pmatrix}$ $\therefore A'(11/4, -5/4), B'(-1, -4), C'(-5, 7)$	M_1 A1A1A1 12	

No. 13

Sketch



B_1 – Correct sketch to include $\bar{AB} = 6cm$, $\bar{BC} = 4{\cdot}8cm$ & $\angle ABC = 150°$

B_1 – Correct drawing without any sketch indicated.

M_1 – Drawing $\bar{AB}$

M_1 – Constructing 60° at B and 30° at B.

Accurate

M_1 – Drawing $\bar{BC}$

M_1 – Drawing $\bar{AD}$

M_1 – Drawing a ⊥lar from D to meet AB at M.

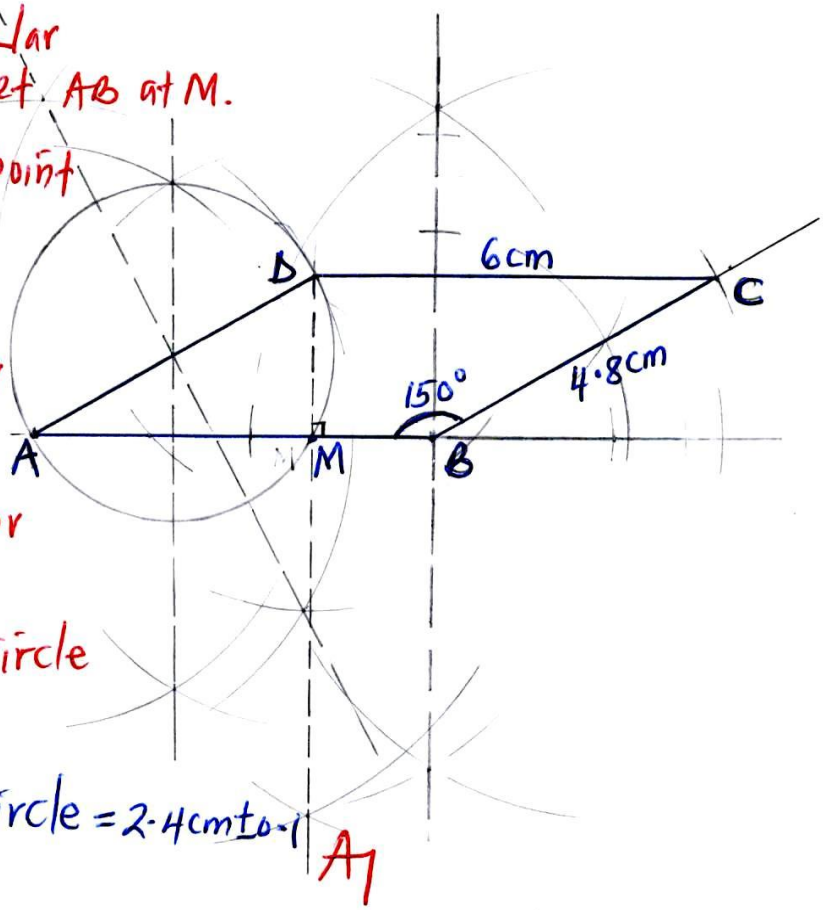
B_1 – for marking point M.

A_1 – $\bar{DM} = 2{\cdot}4cm \pm 0{\cdot}1$

M_1 – 1st ⊥lar bisector of ΔAMD

M_1 – 2nd ⊥ar bisector of ΔAMD

A_1 – Correct circle drawn



C Radius of a circle = $2{\cdot}4cm \pm 0{\cdot}1$ A_1

No.	SOLUTION	MARKS	COMMENTS
14.			

x	-3	-2	-1	0	1	2	3
$2x^2$	18	8	2	0	2	8	18
$-x$	3	2	1	0	-1	-2	-3
-3	-3	-3	-3	-3	-3	-3	-3
y	18	7	0	-3	-2	3	12

B_1 — For all $2x^2$ row correct

B_1 — For all $-x$ row correct

B_2 — For all y-row correct

B_1 — for 4–6 correct

(b) $y = x + 1$

x	0	5
y	1	6

B_1 — for any 2 points stated

(c) (i) $y = 2x^2 - x - 3$

$0 = 2x^2 - 2x - 4$

$y = x + 1$

M_1

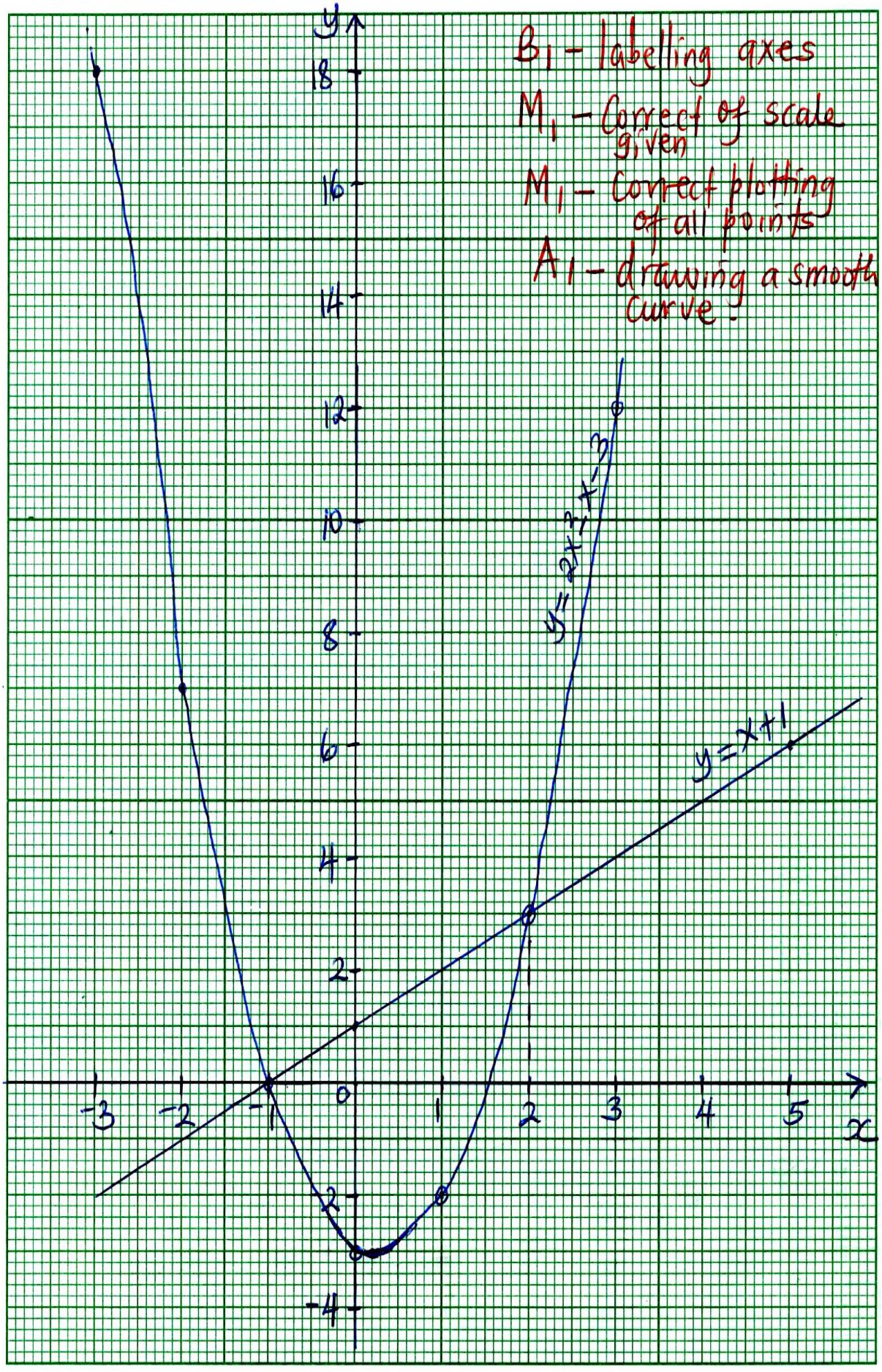
From the graph the line $y = x + 1$ intersect The curve $y = 2x^2 - x - 3$ at $x = -1$ and $x = 2$

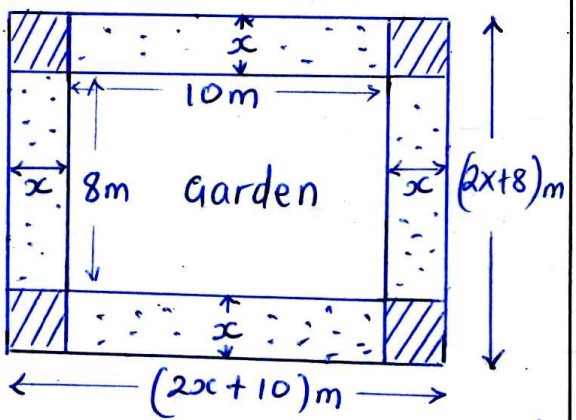
$\therefore$ Solutions to $x^2 - x - 2 = 0$ are $x = -1$ and $x = 2$

A_1 — For both correct x-values

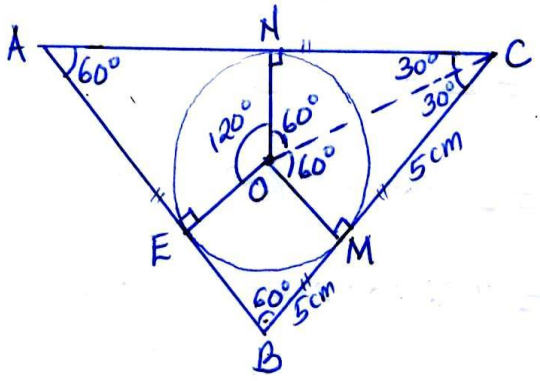
(ii) Minimum value of $y = -3.1$

A_1

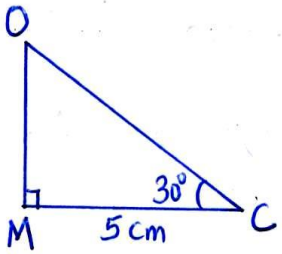


No.	SOLUTION	MARKS	COMMENTS
15-	 Let Area of the garden and its path be A. - Area of the garden be A_1 - Area of the path be A_2 Area, $A = (2x+10)(2x+8)$ $A = (4x^2+36x+80)m^2$	M_1	
	$A_1 = 10 \times 8$ $= 80\ m^2$	M_1	
	$A_2 = 88\ m^2$	M_1	
	But $A = A_1 + A_2$ $4x^2+36x+80 = 80+88$ $4x^2+36x-88=0$ $x^2+9x-22=0$ $(x-2)(x+11)=0$ Either $x = 2\ m$ or $x = -11$	M_1 M_1	

No.	SOLUTION	MARKS	COMMENTS
15.	$\therefore x = 2m$	A1	
	Area of each corner slab $= 2 \times 2$ $= 4m^2$		
	Area of each smaller slab $= 0.5 \times 0.5$ $= 0.25m^2$	M1	
	Area occupied by Big corner slabs $= 4 \times 4$ $= 16m^2$		
	Area covered by smaller slabs $= 88 - 16$ $= 72m^2$	M1	
	Number of smaller slabs $= \frac{72\ m^2}{0.25m^2}$ $= 288$		
	$\therefore$ Total number of slabs $= 288 + 4$ $= 292$	A1	
15 (c)	Cost of corner slabs $= 4 \times 6000$ $= 24,000$		
	Cost of smaller slabs $= 288 \times 5000$ $= 1440,000$	B1	for his/her 24,000 & 1440,000
	$\therefore$ Total cost of covering the path $= 1440,000 + 24000$ $= 1,464,000/=$	M1 A1	
		12	

No.	SOLUTION	MARKS	COMMENTS
16.		B1	

(i) From ΔOMC ; OM = Radius



$\tan 30° = \frac{OM}{5}$ M1

$OM = 5 \times \tan 30°$ M1

$OM = 2.8868$ cm (4d.p)

$\therefore$ Radius of the circle $= 2.8868$cm A1

(ii) Area (sector EON) $= \frac{120}{360} \times \frac{22}{7} \times (2.8868)^2$ M1

$\therefore$ Area of the sector $= 8.7305\ cm^2$ (4d.p) A1

(iii) Area of shaded region = Area of ΔABC − Area of Circle

No.	SOLUTION	MARKS	COMMENTS
16.	Area of $\Delta ABC = \frac{1}{2}ab\sin C$		
	$= \frac{1}{2} \times 10 \times 10 \sin 60^\circ$	M1	
	$= 43.3013 cm^2$ (4dp)	A1	
	Area of circle $= \pi r^2$		
	$= \frac{22}{7} \times (2.8868)^2$	M1	
	$= 26.1914 cm^2$	A1	
	Area of shaded region $= (43.3013 - 26.1914)$	M1	
	$= 17.1099 cm^2$ (4d.p)	A1	

No.	SOLUTION	MARKS	COMMENTS
17.(a).	$x+6 \leq \lvert 15-2x \rvert < 2x+21$		
	for $x+6 \leq 15-2x$		
	$3x \leq 9$	M_1	
	$\therefore x \leq 3$	A_1	
	For $15-2x < 2x+21$		
	$-4x < 6$	M_1	
	$x > -\frac{3}{2}$	A_1	
	$-\frac{3}{2} < x \leq 3$	B_1	
	Integral values of $x = \{-1, 0, 1, 2, 3\}$	A_1	
		06	
(b).	For line 1, Points (0,2) and (3,0)		
	Gradient $= \frac{0-2}{3-0} = -\frac{2}{3}$		
	$\frac{y-2}{x} = -\frac{2}{3}$	M_1	
	$3y-6 = -2x$		
	$2x+3y = 6$		
	$\therefore 2x+3y < 6$	B_1	

No.	SOLUTION	MARKS	COMMENTS
17.	For line 2, points (4,0) and (0,-3) Gradient of $L_2 = \frac{-3-0}{0-4} = \frac{3}{4}$ Equation of L_2: $\frac{y-0}{x-4} = \frac{3}{4}$ $4y = 3x - 12$ $4y - 3x = -12$ $3x - 4y = 12$ $3x - 4y \geq 12$	M_1 B_1	
	For Line 3, points (-2,0) and (0,-3) Gradient of $L_3 = \frac{-3-0}{0+2} = -3/2$ Equation of L_3: $\frac{y-0}{x+2} = \frac{-3}{2}$ $2y = -3x - 6$ $3x + 2y = -6$ $3x + 2y \geq -6$	M_1 B_1	
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